Swarm Extended Kalman Filter

For a nonlinear system: $\dot{x}(t) = F(x,t) + G(x,t)\, w(t)$

$$y(t) = h(x,t) + v(t)$$

with states $x(t) =$

$\dot{v}$		orb frame acceleration scalar
v		orb frame velocity scalar
$\dot{\phi}$		side tilt rate
ϕ		side tilt angle
$\dot{\theta}$		front tilt rate
θ		front tilt angle
ψ		heading
x	=	position East
y	=	position North
x_{ab}	=	x accelerometer bias
y_{ab}		y " "
z_{ab}		z " "
x_{rb}		x rate gyro bias
z_{rb}		z " " "

& measurements $y(t) =$

x_a	x accelerometer
y_a	y "
z_a	z "
x_r	x rate gyro
z_r	z rate gyro
x_g	x GPS
y_g	y "
z_g	z "
ψ_g	heading "
V_g	velocity scalar from GPS
ω	wheel speed

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Nonlinear System $\dot{x}(t) = F(x,t) + G(x,t)w(t)$

$F(x,t) =$

$\ddot{v} = 0$

$\dot{v} = \dot{v}$

$\ddot{\phi} = 0$

$\dot{\phi} = 0$

$\ddot{\theta} = 0$

$\dot{\theta} = \dot{\theta}$

$\dot{\psi} = \frac{v}{r} \tan \phi$

$\dot{x} = v \cos \psi$

$\dot{y} = v \sin \psi$

$\dot{x}_{ab} = 0$

$\dot{y}_{ab} = 0$

$\dot{z}_{ab} = 0$

$\dot{x}_{rb} = 0$

$\dot{z}_{rb} = 0$

Measurements $y(t) = h(x,t) + v(t)$

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = {}^{C}_{A}R \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ -g \end{bmatrix}$$

where $${}^{C}_{A}R = \begin{bmatrix} c\psi c\theta + s\psi s\phi s\theta & -s\psi c\theta + c\psi s\phi s\theta & -c\phi s\theta \\ s\psi c\theta & c\psi s\theta & s\phi \\ c\psi s\theta - s\psi s\phi c\theta & -s\psi s\theta - c\psi s\phi c\theta & c\phi c\theta \end{bmatrix}$$

$$\ddot{x} = \dot{v}\cos\psi - \frac{v^2}{r}\tan\phi \sin\psi$$

$$\ddot{y} = \dot{v}\sin\psi + \frac{v^2}{r}\tan\phi \cos\psi$$

$$g = 9.81\, m/s^2$$

no sensor $\longrightarrow$ $$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + {}^{C}_{B}R \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + {}^{C}_{A}R \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

where $${}^{C}_{B}R = \begin{bmatrix} c\theta & s\phi s\theta & -c\phi s\theta \\ 0 & c\phi & s\phi \\ s\theta & -s\phi c\theta & c\phi c\theta \end{bmatrix}$$

so $x_r = c\theta\,\dot{\phi} - c\phi s\theta\,\dot{\psi}$

$z_r = s\theta\,\dot{\phi} + c\phi c\theta\,\dot{\psi}$

GPS $x_g = x$

$y_g = y$

$\psi_g = \psi$

$v_g = v$

Encoder $\omega = \dfrac{v}{r\cos\phi}$

Note: $c\theta$ means $\cos\theta$, $s\theta$ means $\sin\theta$, etc.